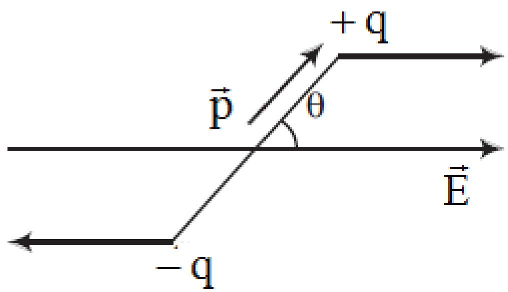
PREVIOUS YEAR CBSE FOREIGN 2015

1. Write the expression for the torque τ acting on a dipole of dipole moment ***p*** placed in an electric field ***E*** [1]

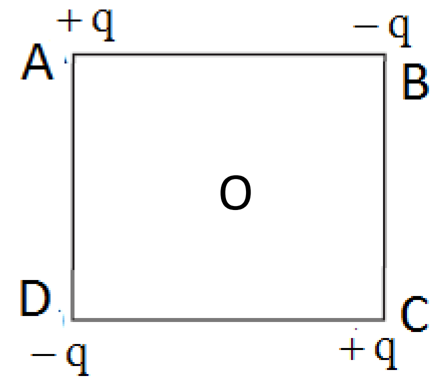
Torque acting on the dipole, placed in uniform electric field is given by

$$\vec{\tau} = \vec{p} \times \vec{E}$$



11. Four charges $+q$, $-q$, $+q$ and $-q$ are to be arranged respectively at the four corners of a square *ABCD* of side '*a*'. (*a*) Find the work required to put together this arrangement. (*b*) A charge q_0 is brought to the centre of the square, the four charges being held fixed. How much extra work is needed to do this? [3]

(a)



$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA} + W_{AC} + W_{BD}$$

$$W_{AB} = W_{BC} = W_{CD} = W_{DA} = \frac{k.q.(-q)}{a}$$

$$W_{AC} = W_{BD} = \frac{kq.q}{a\sqrt{2}}$$

$$W = 4.\frac{-kq^2}{a} + 2.\frac{kq^2}{a\sqrt{2}} = \frac{kq^2}{a}\left(\sqrt{2} - 2\right)$$

(b)

$$W = q_0.\Delta V = q_0(V_O - V_\infty)$$

$$AO = BO = CO = DO = \frac{a}{\sqrt{2}}$$

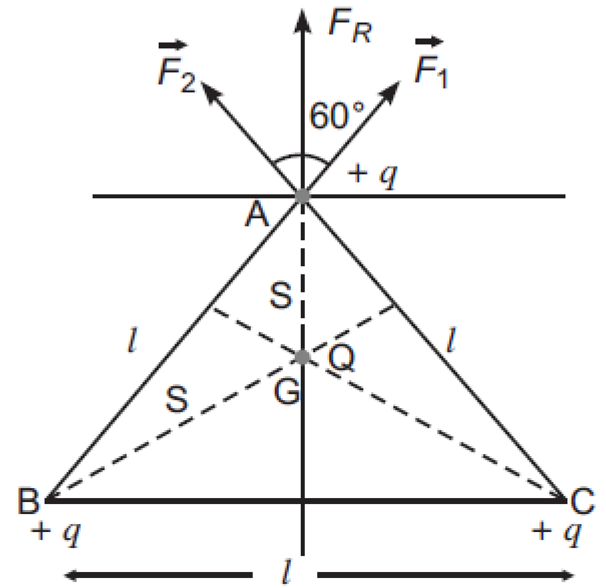
$$V_O = \frac{kq}{AO} - \frac{kq}{BO} + \frac{kq}{CO} - \frac{kq}{DO} = 0$$

$$W = q_0 \times 0, \quad \because V_\infty = V_O = 0$$

OR

Three point charges +*q* each are kept at the vertices of an equilateral triangle of side '*l*'. Determine the magnitude and sign of the charge to be kept at its centroid so that the charges at the vertices remain in equilibrium. [3]

The charge, at any one vertex will remain in equilibrium, if the net electric force at that point, due to the other three charges, is zero.



Let Q be the required charge.

$\vec{F}_1$ = Force at A due to the charge at B,

$$\vec{F}_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{l^2} \text{ along } \overrightarrow{BA}$$

Force at A due to the charge at C,

$$\vec{F}_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{l^2} \text{ along } \overrightarrow{CA}$$

$$\vec{F}_1 + \vec{F}_2 = \sqrt{3} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{l^2} \text{ along } \overrightarrow{GA}$$

$$\text{Centroid at } \frac{l}{\sqrt{3}} \text{ from } A$$

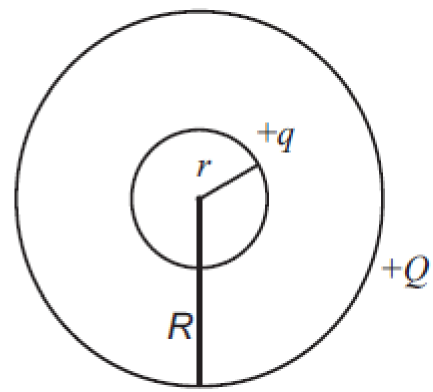
$$\text{Force at A due to charge at } G = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Qq(3)}{l^2}$$

$$\therefore \qquad 3Qq = -\sqrt{3}q^2$$

$$Q = -\frac{q}{\sqrt{3}}$$

25. (*a*) A small conducting sphere of radius '*r*' carrying a charge $+q$ is surrounded by a large concentric conducting shell of radius R on which a charge $+Q$ is placed. Using Gauss's law, derive the expressions for the electric field at a point '*x*'. (*i*) between the sphere and the shell ($r < x < R$), (*ii*) outside the spherical shell. (*b*) Show that if we connect the smaller and the outer sphere by a wire, the charge q on the former will always flow to the latter, independent of how large the charge Q is. [5]

(a) For Gaussian surface of radius, $r < x < R$



$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \qquad |E| \oint |dS| = \frac{q}{\varepsilon_0}$$

$$|E| \cdot 4\pi x^2 = \frac{q}{\varepsilon_0}$$

Hence, $$|E| = \frac{1}{4\pi x^2} \frac{q}{\varepsilon_0}$$

For Gaussian surface of radius $x > R$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{S}} = \frac{(q + Q)}{\varepsilon_0}$$

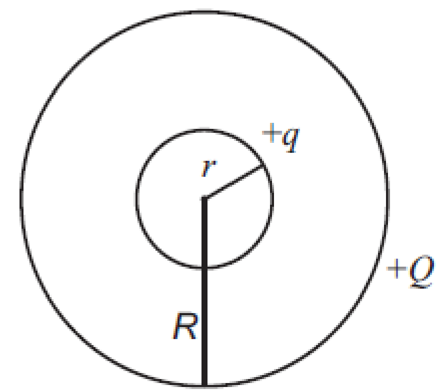
$$|E| \cdot 4\pi x^2 = \frac{(q + Q)}{\varepsilon_0}$$

Hence, $$|E| = \frac{1}{4\pi \varepsilon_0} \cdot \frac{(q + Q)}{x^2}$$

(*b*) The electric potential of the inner sphere is

$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

and electric potential of the outer shell



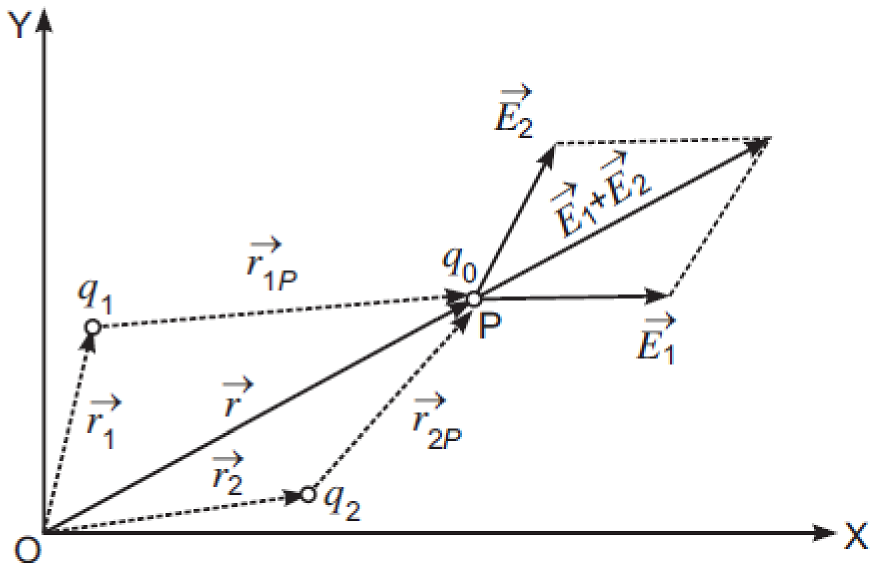
$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}$$

$$V_1 - V_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right) > 0$$

Since $V_1 > V_2$, so charge will always flow from the smaller sphere to the larger sphere.

OR

(a) Consider a system of n charges q_1, q_2, ... q_n with position vectors r_1 , r_2 , r_3 , ... r_n relative to some origin 'O'. Deduce the expression for the net electric field E at a point P with position vector r_p , due to this system of charges. (b) Find the resultant electric field due to an electric dipole of dipole moment, $2aq$, ($2a$ being the separation between the charges $\pm q$) at a point distant 'x' on its equator. [5]



Consider a system of N point charges $q_1, q_2 .... q_N$ having position vectors $\vec{r}_1, \vec{r}_2 ... \vec{r}_N$ with respect to origin O. We have to find electric field at a point whose position vector is $\vec{r}$.

Force on q_0 kept at P due to charge q_1 is given as

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_0}{r_{1P}^2} \hat{r}_{1P}$$

where $\hat{r}_{1P}$ is a unit vector in the direction from q_1 to P and r_{1P} is the distance between q_1 and P.

Hence the electric field at point P due to charge q_1 is

$$\vec{E_1} = \frac{\vec{F_1}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{1P}^2} \hat{r}_{1P}$$

Similarly, electric field at P due to charge q_2 is

$$\vec{E_2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}^2} \hat{r}_{2P}$$

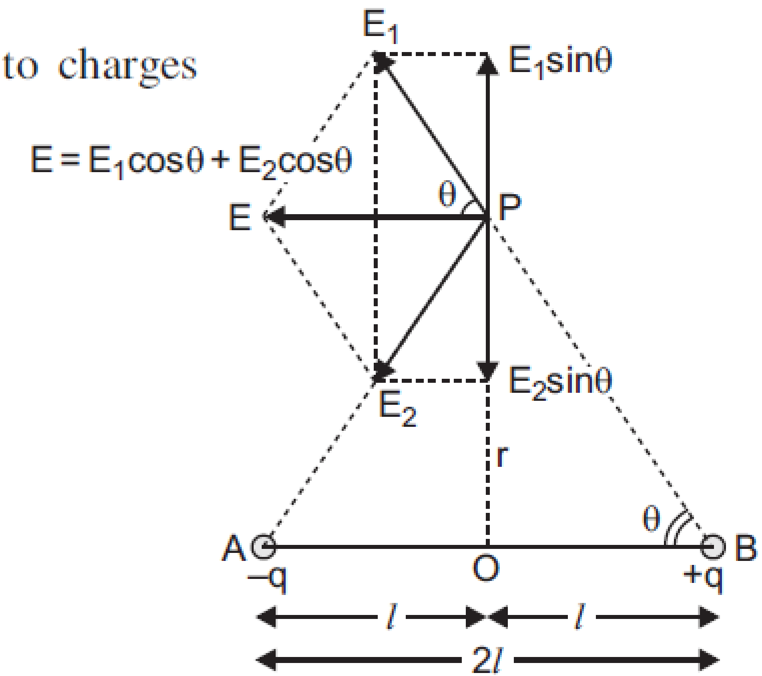
By principle of superposition , net electric field is given as vector sum of all electric fields

Hence, the electric field at point P due to the system of N charges is

$$\vec{E} = \vec{E_1} + \vec{E_2} + \ldots + \vec{E_N}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_{1P}^2}\hat{r}_{1P} + \frac{q_2}{r_{2P}^2}\hat{r}_{2P} + \ldots + \frac{q_N}{r_{NP}^2}\hat{r}_{NP}\right] = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_{iP}^2}\hat{r}_{iP}$$

At a point of equatorial line: Consider a point P on broad side on the position of dipole formed of charges $+q$ and $-q$ at separation $2l$. The distance of point P from mid point (O) of electric dipole is r. Let $\vec{E_1}$ and $\vec{E_2}$ be the electric field strengths due to charges $+q$ and $-q$ of electric dipole.



From fig. $AP = BP = \sqrt{r^2 + l^2}$

$$\therefore \quad \vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + l^2} \text{ along } B \text{ to } P$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2 + l^2} \text{ along } P \text{ to } A$$

Clearly $\vec{E_1}$ and $\vec{E_2}$ are equal in magnitude *i.e.* $|\vec{E_1}| = |\vec{E_2}|$ or $E_1 = E_2$

To find the resultant of $\vec{E_1}$ and $\vec{E_2}$, we resolve them into rectangular components.

Component of $\vec{E_1}$ parallel to $AB = E_1 \cos\theta$, in the direction to $\vec{BA}$

Component of $\vec{E_1}$ perpendicular to $AB = E_1 \sin\theta$ along OP

Component of $\vec{E}_2$ parallel to $AB = E_2 \cos\theta$, in the direction $\overrightarrow{BA}$

Component of $\vec{E}_2$ perpendicular to $AB = E_2 \sin\theta$ along PO

$\therefore$ Resultant electric field at P is $E = E_1 \cos\theta + E_2 \cos\theta$

But $E_1 = E_2 = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q}{(r^2 + l^2)}$

$$\cos\theta = \frac{OB}{PB} = \frac{l}{\sqrt{r^2 + l^2}} = \frac{l}{(r^2 + l^2)^{1/2}}$$

$$E = 2E_1 \cos\theta = 2 \times \frac{1}{4\pi\varepsilon_0} \frac{q}{(r^2 + l^2)} \cdot \frac{l}{(r^2 + l^2)^{1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2ql}{(r^2 + l^2)^{3/2}}$$

But $q.2l = p$ = electric dipole moment

$$\therefore E = \frac{1}{4\pi\varepsilon_0} \frac{p}{(r^2 + l^2)^{3/2}}$$