A rigid body is defined as that body which does not undergo any change in shape or volume when external forces are applied on it.

A point in the system at which whole mass of the body is supposed to be concentrated is called **centre of mass** of the body.

For a system consisting of n particles with masses $m_1, m_2, m_3 \ldots m_n$ with position vectors $r_1, r_2, r_3 \ldots r_n$, the total mass of the system is, $M = m_1 + m_2 + m_3 + \ldots\ldots\ldots\ldots + m_n$

$$X = \frac{\sum m_i x_i}{M};\ Y = \frac{\sum m_i y_i}{M};\ Z = \frac{\sum m_i z_i}{M}$$

Angular displacement ($\theta$) : Unit is radian and dimension is $[M^0L^0T^0]$

Angular velocity ($\omega$): Unit is radian/sec and dimension is $[M^0L^0T^{-1}]$

$$\text{Angular velocity } (\omega) = \frac{d\theta}{dt} \qquad \text{Angular acceleration } (\alpha) = \frac{d\omega}{dt}$$

$$\omega = \omega_0 + \alpha t;\quad \omega^2 = \omega_0^2 + 2\alpha\theta;\quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

**Moment of inertia:** The moment of inertia (I) of a rigid body about an axis is defined as the sum of product of masses of all particles of the body and the squares of their perpendicular distances from the axis of rotation.

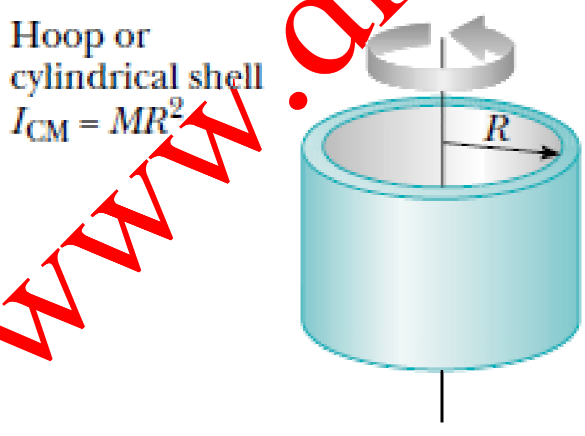
**Unit:** kg $m^2$ **Dimensions** : $[ML^2T^0]$

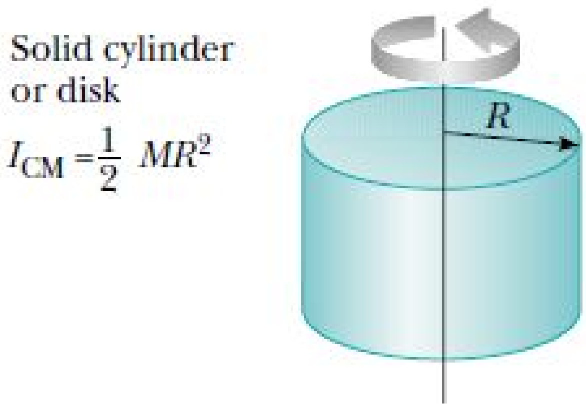
$$\text{Mathematically } I = \sum m_i r_i^2$$

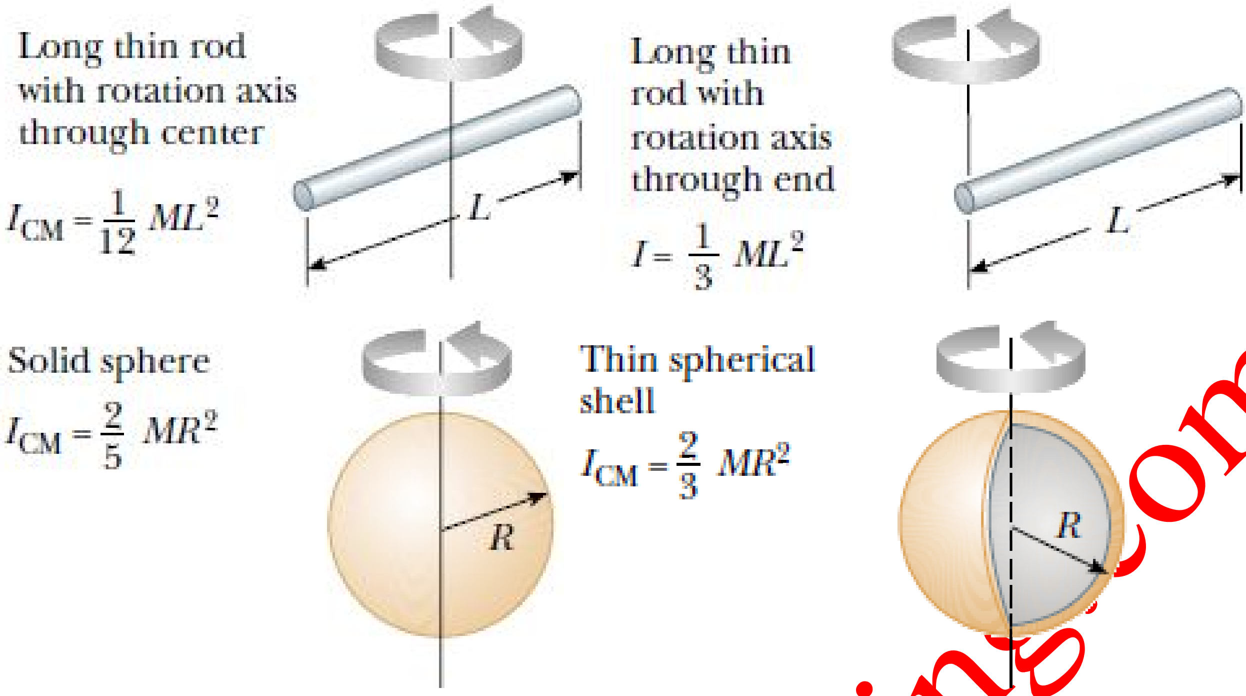
Where $m_i$ is the mass of each particle and $r_i$ is the relative distance of the particle from the axis of rotation

**Radius of gyration**: Distance from axis where if all mass is kept, It would have same moment of inertia **Unit:** m **Dimensions** : $[ML^1T^0]$

$$\text{Mathematically } K = \sqrt{\frac{r_1^2 + r_2^2 + \ldots\ldots + r_n^2}{n}}$$

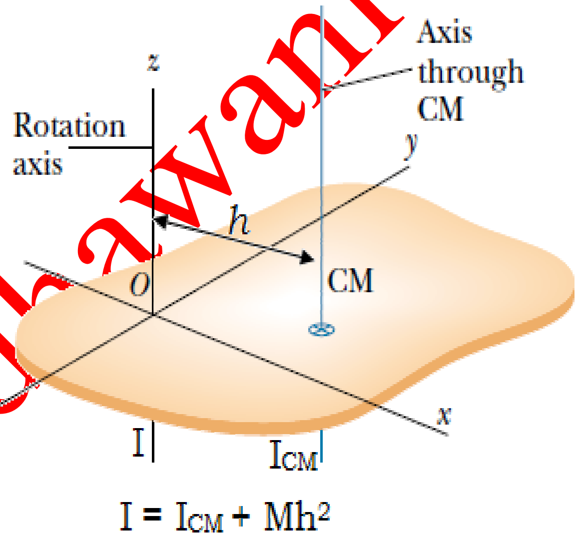


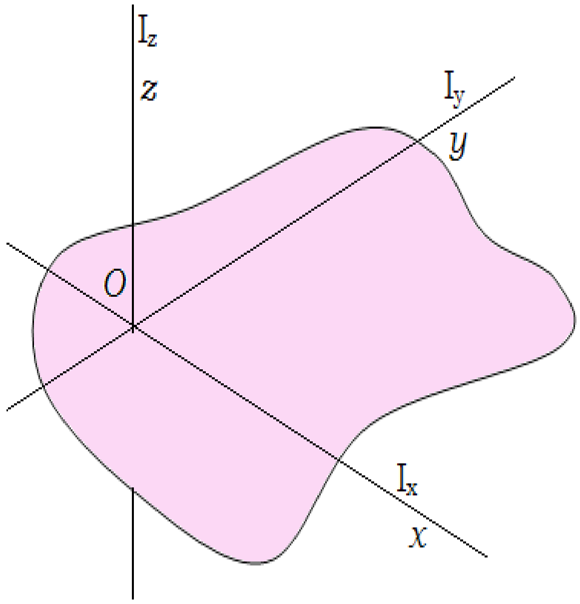



THEOREMS to FIND MOMENT OF INERTIA

PARALLEL AXIS :The moment of inertia of a body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of gravity and the product of the mass of the body and the square of the distance between the two axes.




PERPENDICULAR AXIS The moment of inertia of a plane laminar body about an axis perpendicular to the plane is equal to the sum of the moments of inertia about two mutually perpendicular axes in the plane of the lamina such that the three mutually perpendicular axes have a common point of intersection.




$I_z = I_x + I_y$

| Rotational Motion About a Fixed Axis | Linear Motion |
|---|---|
| Angular speed $\omega = d\theta/dt$ | Linear speed $v = dx/dt$ |
| Angular acceleration $\alpha = d\omega/dt$ | Linear acceleration $a = dv/dt$ |
| Resultant torque $\Sigma\tau = I\alpha$ | Resultant force $\Sigma F = ma$ |
| If $\alpha$ = constant $\begin{cases} \omega = \omega_0 + \alpha t \\ \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 = \omega_0^2 + 2\alpha\theta \end{cases}$ | If $a$ = constant $\begin{cases} v = u + at \\ s = ut + \frac{1}{2}at^2 \\ v^2 = u^2 + 2as \end{cases}$ |
| Work $W = \int_{\theta_i}^{\theta_f} \tau\, d\theta$ | Work $W = \int_{x_i}^{x_f} F_x\, dx$ |
| Rotational kinetic energy $K_R = \frac{1}{2}I\omega^2$ | Kinetic energy $K = \frac{1}{2}mv^2$ |
| Power $\mathcal{P} = \tau\omega$ | Power $\mathcal{P} = Fv$ |
| Angular momentum $L = I\omega$ | Linear momentum $p = mv$ |
| Resultant torque $\Sigma\tau = dL/dt$ | Resultant force $\Sigma F = dp/dt$ |



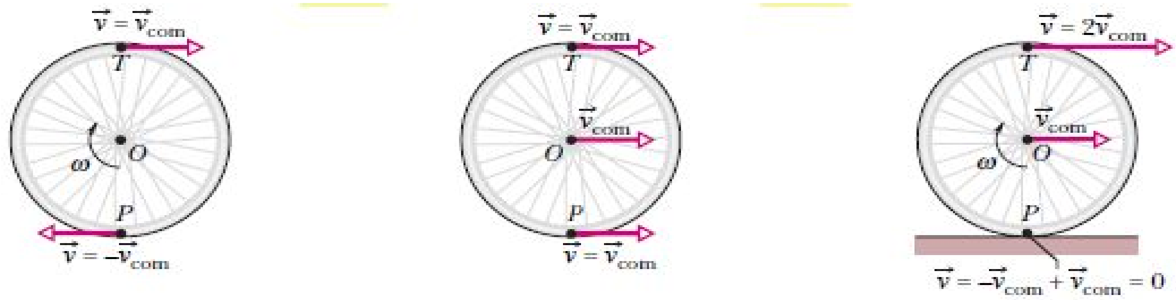
ROLLING : when center of mass has translational motion, and rotational about center of mass.

REMEMBER that ROLLING is not possible without friction. BUT this frictional force does not do ANY WORK. So loss of frictional WORK is not taken into account.

FOR PERFECT rolling $V_{CM} = R\,\omega$

If $V_{CM} > R\omega$, then example is skidding of vehicle

And $V_{CM} < \omega R$ is example when car is not able to move on either SAND or ICE



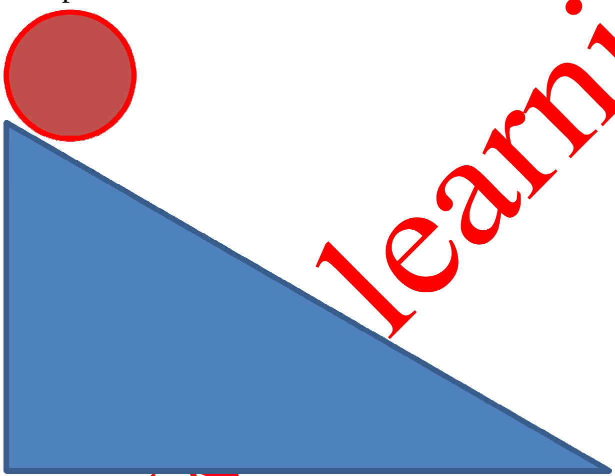

CONSERVATION of ANGULAR MOMENTUM: when NO external TORQUE is acting. Torque is also rate of change of ANGULAR MOMENTUM. It has same role in ROTATIONAL MOTION as force in translational Motion

$$\tau = \frac{dL}{dt} = \frac{d(\vec{r} \times \vec{p})}{dt}$$

$$\text{TORQUE} = \text{Force} \times \perp_r \text{Distance}$$

$$\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \tau = rF\sin\theta$$

For body rolling down an inclined plane



In this case please do remember that, we require minimum amount of friction, so minimum coefficient of friction is required. If we have available friction coefficient, then rolling is possible

$$\mu_{MIN} = \frac{\tan\theta}{1+\frac{MR^2}{I}} \qquad a = \frac{g\sin\theta}{1+\frac{I}{MR^2}} \qquad f_{FRICTION} = \frac{Mg\sin\theta}{1+\frac{I}{MR^2}}$$

$$a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}} \qquad v = \frac{2gH}{1+\frac{K^2}{R^2}} \qquad t = \frac{2s\left(1+\frac{K^2}{R^2}\right)}{g\sin\theta}$$

In all these formulas K is radius of GYRATION

These all values are calculated by using conservation of energy

For perfectly ROLLNG body $MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$