Electric flux: is defined as total number of electric field lines passing from any area, denoted by ϕ_E or ϕ mathematically it is defined as integral of **E** and d**S** $\Rightarrow$ $\phi = \int \mathbf{E}.d\mathbf{S}$ with units of Newton meter^2/Coulomb

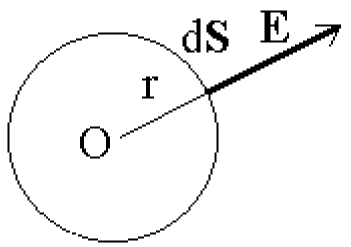
Remember area vector ***S*** *is perpendicular to the surface of area under consideration.*

Gauss Law: states that surface integral of electric field produced by any source over any closed surface **S** enclosing a volume V is $1/\varepsilon_0$ times total charge contained inside surface.

It is to note that electric charges outside Gaussian surface do not contribute to electric field.

Derivation of Coulomb law, E due to linear charged & E due to a charged sphere (Hollow/Conductor/uniformly Charged)

Basics: in all the derivations we get a Gaussian surface symmetrical to charge. Find out direction of **E** and **S** at various points on surface. Whenever **E** is perpendicular to surface **E**.d**S** is zero and if **E** is || to **S** **E.dS** becomes **E**d**S.** After calculating flux we find the charge in given Gaussian surface. And use $\int \mathbf{E}.d\mathbf{S} = Q/\varepsilon_0$



In 1st case we take point charge Q placed at center, then taking sphere as Gaussian surface (radius r) we draw as shown in Figure 1.Considering point 'A' near the surface we see that '**E**' and 'd**S**' are in same direction thus

$$\phi = \oint E.dS = \oint E.dS\cos\theta$$

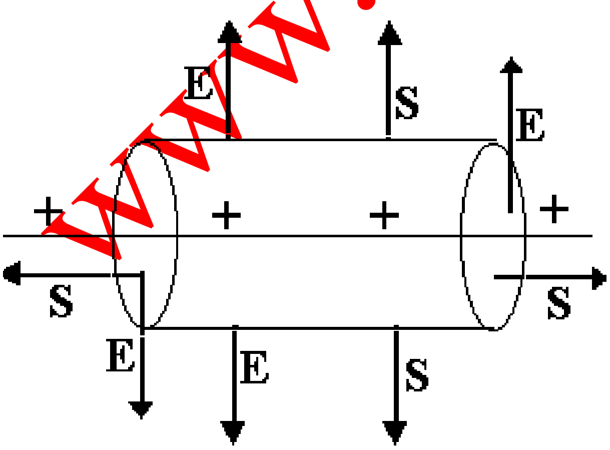
$$= \oint E.dS = E\oint dS \quad (\text{E is constant})$$

$$= E.4\pi r^2$$

Now as per Gauss law it should be equal to charge contained in volume i.e. Q. equating both we get $\phi = E.4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

If we place charge Q' at point A the force is given by **F** = Q'**E** placing the value for **E** gives $F = \frac{1}{4\pi\varepsilon_0}\frac{QQ'}{r^2}$ which is Coulomb's law.

In integration we have taken E out of the integral with assumption that due to symmetry each point is same distance from center, thus same E



In Second case for a linear conductor (λ Coulomb/meter) charge we take Gaussian surface as a cylinder (radius r & length L) with axis collinear with the charge.

Drawing different vectors at different places for **E** and **S** we see that at the curved surface **E** and **S** are parallel to each other, while at plain surfaces they are perpendicular to each other.
Thus only **E** & **S** which are on curved surface contribute to the calculation of flux. Mathematically

$$\phi = \oint E.dS = \int_{CSA} E.dS + \int_{Ends} E.dS$$

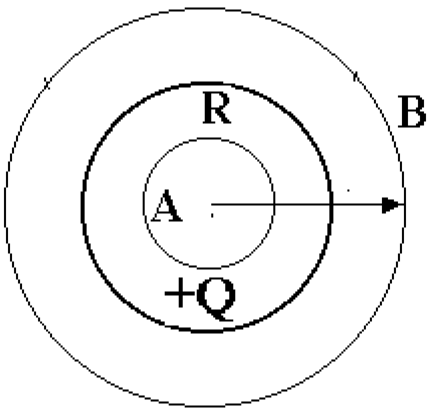
$$= \int_{CSA} EdSCos90^0 + \int_{Ends} EdSCos0^0$$

$$= \int_{Ends} EdS = E\int_{Ends} dS \ \ (E\ Constant)$$

$$= E.2\pi rL$$

Now this flux is also equal to charge in the cylinder = λL. Equating these we get

$$E2\pi rL = \frac{\lambda L}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi r\varepsilon_0}$$



In this case we try to get **E** due to hollow or charged conductor(radius R) In both the cases charge is residing at the periphery. So we take 3 different guassian surfaces at distance 'r' from the center of sphere
(i) r < R sphere 'A'
(ii) r = R sphere 'R'
(iii) r > R sphere 'B'

Case (i) $\phi = \oint E.dS = \frac{Q}{\varepsilon_0}$ but since charge in sphere 'A' is zero. Thus as per Gauss law $\phi = Q/\varepsilon_0 = 0$. Equating flux from both we get

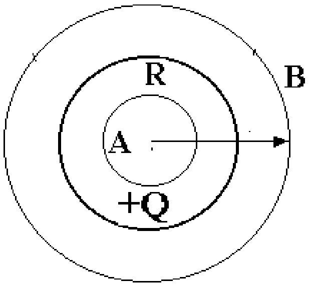
$$\phi = \oint E.dS = \frac{Q}{\varepsilon_0} = 0$$ as $dS \neq 0 \Rightarrow E = 0$

Case(ii) $\phi = \oint E.dS = E\oint dS$ (E constant)

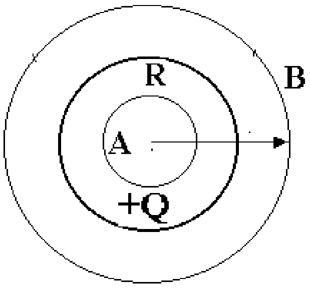
$$\phi = E.4\pi R^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

Case (iii) $\phi = \oint E.dS = E\oint dS$ also

$$\phi = \frac{Q}{\varepsilon_0} = E.4\pi r^2 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$



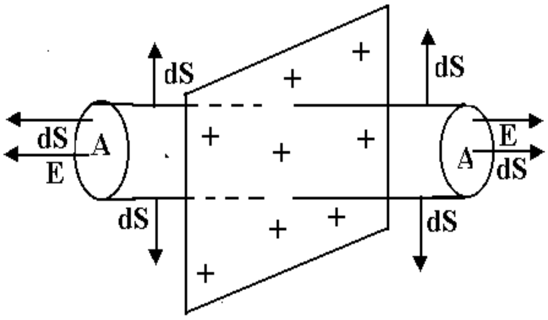
Lastly we take derivation of uniformly charged sphere, where charge 'Q' is uniformly distributed over the full sphere.



Obviously it has to be insulator. Thus the derivation for $r \geq R$ is same as for conductor (hollow or solid sphere). Charge per unit volume(ρ) = $Q/V = 3Q/4\pi R^3$
The only change is in the **E** inside the sphere, as in this case some charge is there, while in previous cases there was no charge. Applying Gauss in this case

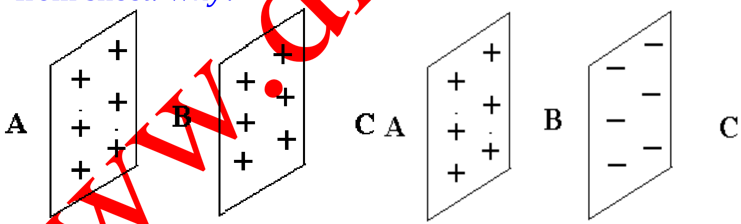
Which is equal to charge in Guassian sphere

Thus $\phi = \frac{Qr^3}{\varepsilon_0 R^3} \Rightarrow E.4\pi r^2 = \frac{Qr^3}{\varepsilon_0 R^3} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Qr}{R^3}$

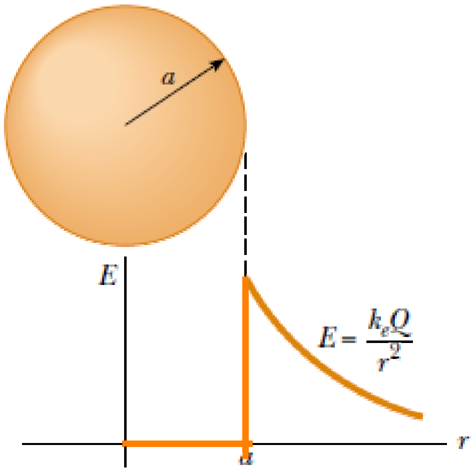


Considering the case of infinitely large dimensioned charged sheet having charge per unit area as σ Coul.m^2. We take cylindrical Guassian surface with axis perpendicular to the plane of sheet. It is clear that at curved surface are **E** and d**S** are perpendicular to each other. Thus CSA does not contribute to flux, but only circular sheet at the extreme ends contribute to flux as **E** and d**S** are parallel in this case.

$\phi = \oint E.dS = E.2A = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$ It is quiet interesting to note that in present case **E** is independent of distance from sheet. *Why*?



E due to Charged Spherical SHELL/CONDUCTING SPHERE



Electric field due to Charged NON CONDUCTING SPHERE