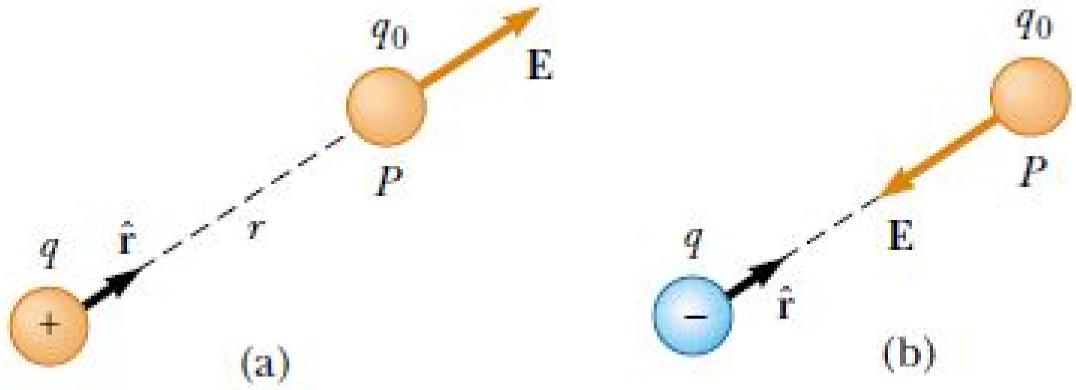
Electric Field Intensity

$$E = \underset{q_0 \to 0}{Lt} \frac{F}{q_0}$$

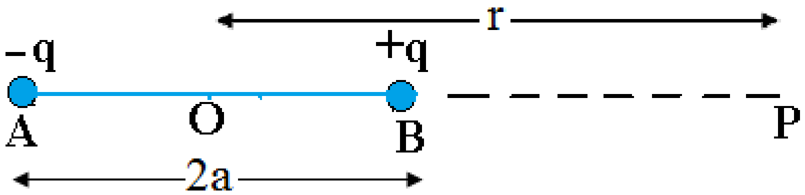


$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2}; \qquad \text{BUT } E = \underset{q_0 \to}{Lt} \cdot \frac{F}{q_0} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r^2} \frac{1}{q_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

For more than ONE charges principle of Superposition is applied

DIPOLE AXIAL LINE



$$E_+ = k\frac{q}{(r-a)^2} \quad \text{towards} + \text{ve x axis}$$

$$E_- = k\frac{q}{(r+a)^2} \quad \text{towards} - \text{ve x axis} \qquad \vec{E} = \vec{E}_+ + \vec{E}_- \Rightarrow E_{NET} = E_+ - E_-$$

$$E_{NET} = k\frac{q}{(r-a)^2} - k\frac{q}{(r+a)^2} \quad \text{towards} + \text{ve x axis}$$

$$= kq\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] = kq\left[\frac{4ar}{\left(r^2-a^2\right)^2}\right] \qquad \vec{E} = \frac{2kp\vec{r}}{\left(r^2-a^2\right)^2}$$

$$\text{For } r >> a \Rightarrow r^2 - a^2 \approx r^2 \qquad \vec{E} = \frac{2kp\vec{r}}{r^4} \Rightarrow E = \frac{2kp}{r^3}$$

DIPOLE EQUATORIAL LINE

$$\angle ABC = \theta = \angle CBA$$

$$E_+ = E_- = k\frac{q}{r^2 + a^2}$$

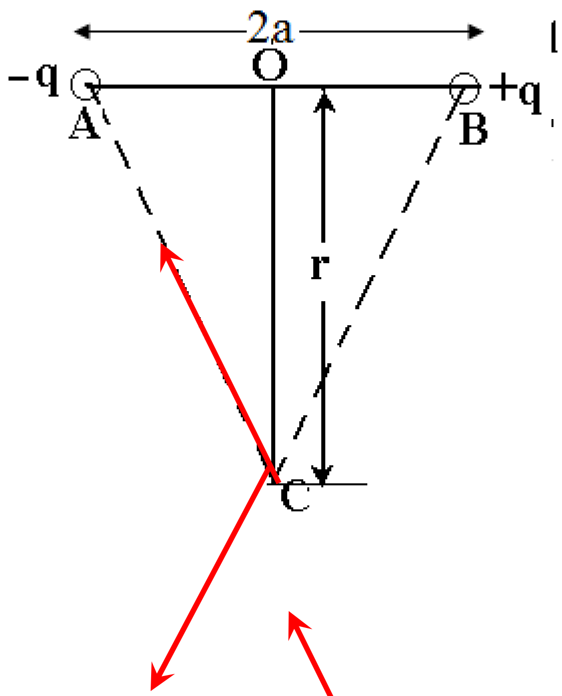
$$\text{Horizontal component} = E\cos\theta$$

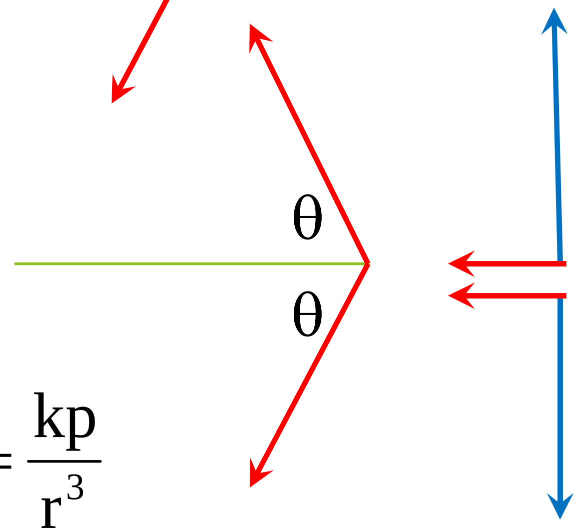
$$\text{Vertical Component} = E\sin\theta$$

$$\text{NET FIELD} = 2E\cos\theta = 2k\frac{q}{r^2 + a^2}.\cos\theta$$

$$E = 2k\frac{q}{r^2 + a^2}.\frac{a}{\sqrt{r^2 + a^2}} = k\frac{2qa}{(r^2 + a^2)^{3/2}}$$

$$E = \frac{kp}{(r^2 + a^2)^{3/2}} \quad \text{For } r >> a; r^2 + a^2 \approx r^2$$





$$E = \frac{kp}{r^3}$$

DIPOLE EQUATORIAL LINE

$$E_+ = E_- = k\frac{q}{r^2 + a^2} = E(\text{say})$$

Resultant

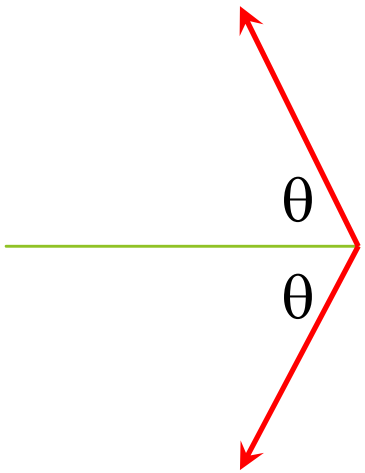
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$E_{NET} = \sqrt{E^2 + E^2 + 2E^2\cos 2\theta}$$

$$E_{NET} = E\sqrt{2(1+\cos 2\theta)}$$

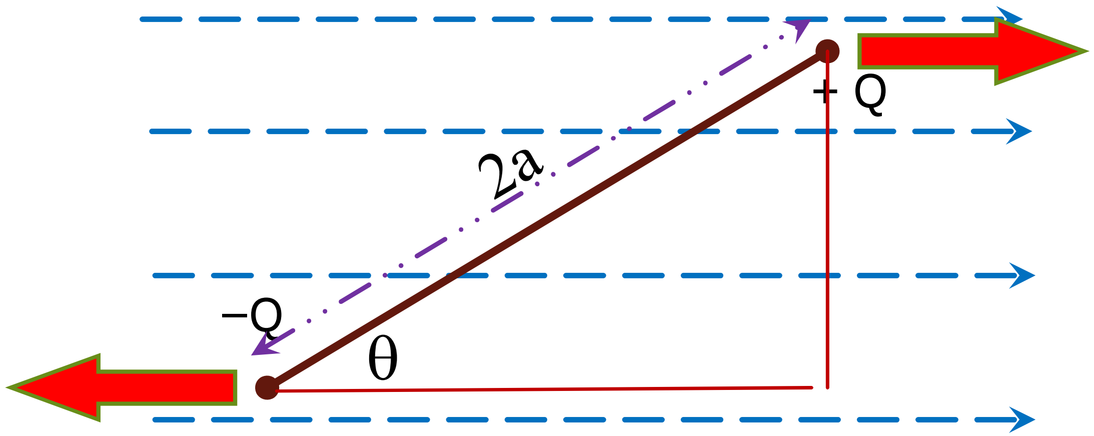
Recall $\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow 2\cos^2\theta = 1 + \cos 2\theta$

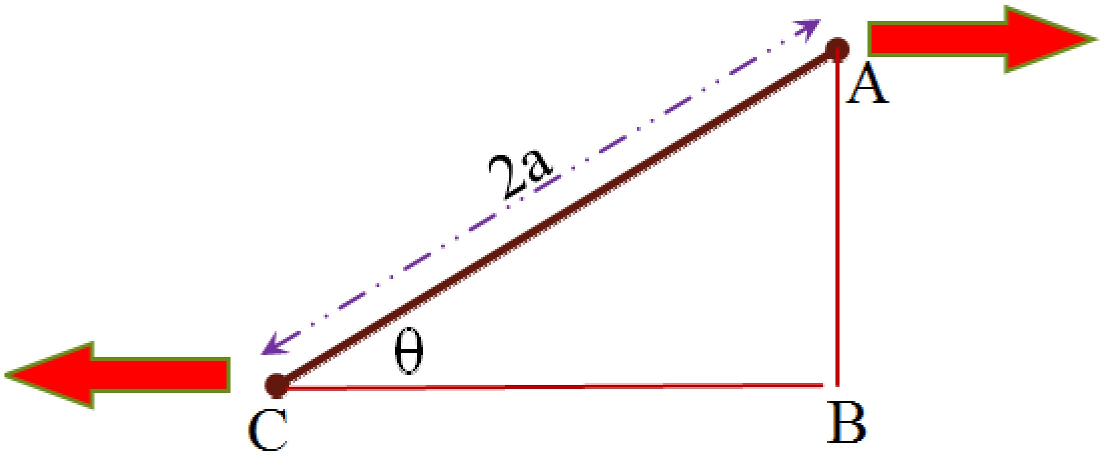
$$E_{NET} = E\sqrt{2\times 2\cos^2\theta} = 2E\cos\theta$$



$$E_{NET} = 2k\frac{q}{r^2 + a^2}\cdot\frac{a}{\sqrt{r^2 + a^2}} = \frac{kp}{(r^2 + a^2)^{3/2}}$$

DIPOLE in UNIFORM FIELD





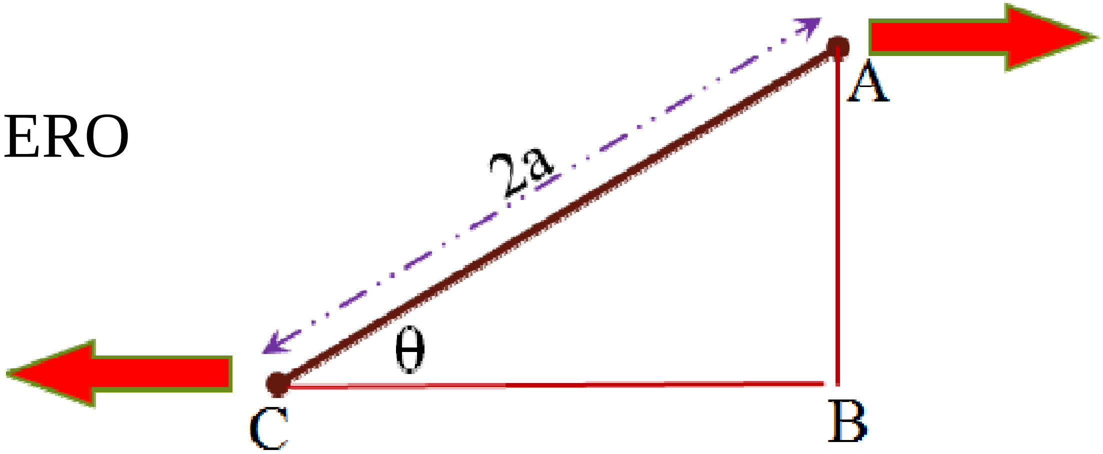
TORQUE on DIPOLE

$\text{Net Force} = QE - QE = 0$

Uniform Field Net F is ZERO

Torque about C point

$\text{TORQUE} = F \times \perp \text{distance}$

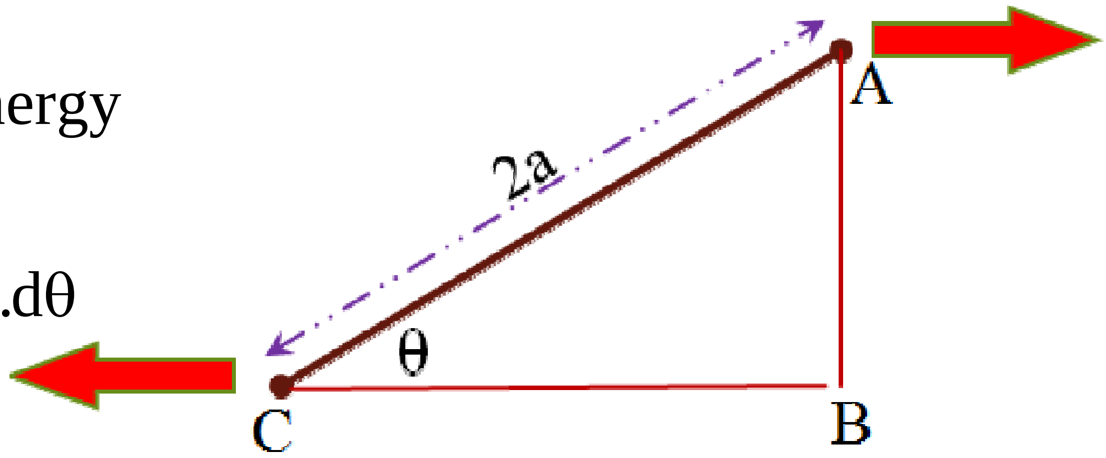


$$\frac{AB}{AC} = \sin\theta \Rightarrow AB = AC\sin\theta = 2a\sin\theta$$

$$\tau = 2aQ \times E\sin\theta = pE\sin\theta \Rightarrow \vec{\tau} = \vec{p} \times \vec{E}$$

$$\text{when } \theta = 90° \Rightarrow \tau_{MAX} = pE \quad \text{and} \quad \text{when } \theta = 0°/180° \Rightarrow \tau_{MIN} = 0$$

ENERGY of DIPOLE in E



$$\text{Work done} = \text{Potential Energy}$$

$$\text{Work done} = \int F.ds = \int \tau.d\theta$$

$$U = \int_{\theta_2}^{\theta_1} pE\sin\theta.d\theta = -pE\cos\theta\Big|_{\theta_2}^{\theta_1} \; -pE(\cos\theta_1 - \cos\theta_2)$$

$$U = -pE\cos\theta = -\vec{p}.\vec{E}$$

$$U_{MIN} = -pE \text{ at } \theta = 0^\circ \qquad \theta = 0^\circ \Rightarrow \text{Stable Equilibrium}$$

$$U_{MAX} = pE \text{ at } \theta = 180^\circ \qquad \theta = 180^\circ \Rightarrow \text{Unstable Equilibrium}$$